Power in mixed exchange networks: a rational choice model

Kazuo Yamaguchi

NORC / The University of Chicago, Chicago, IL 60637, USA

Abstract


1. Introduction

Coleman (1973, 1990) introduced a mathematical model of collective action where actors maximize utility by exchanging their resources. One major concern he addresses using this model was the distribution of power, the total value of resources actors acquire through exchange, among actors at the equilibrium of exchange. This paper extends the formal model and theoretical approach developed by Coleman.

More generally, Coleman stated in his essay "A Rational Choice Perspective on Economic Sociology" (1994, p. 116) that

Rational choice theory is not theory designed to account for action, despite its name. It is theory designed to account for the functioning of social and economic systems. Yet it explains the functioning not merely remaining at the level of the system but by actions of individuals who make up the system, together with linkages between these actions and the level of the systems.
Contrast this characterization with Elster’s (1986, p. 1) statement in the introduction to *Rational Choice*, which he edited:

The theory of rational choice is, before it is anything else, a normative theory. It tells us what we ought to do in order to achieve our aims as well as possible.

One major difference between the two theorists is that Elster is concerned with the mechanism of individual choices and the basis of rationality, such as rationality of belief in relation to some objective evidence and the rationality of collecting that evidence, while Coleman was mainly concerned with the collective outcome of individual choices at the societal level, rather than the individual choices themselves, and did not address the basis of rationality. This *choice* by Coleman in building rational choice theory reflects his preference for simplification, or parsimony, in the model of action, and his concern with the process through which individual choices yield a collective outcome under social structural constraints on the mechanism of such a *micro–macro* link. From this perspective, Coleman (1986) advocated that theories about *macro–macro* links be constructed by combining *macro–micro* and *micro–macro* links, emphasizing the importance of the latter link, i.e., the mechanism of aggregating individual choices to generate a collective outcome, which he considered to be underdeveloped in sociology.

This paper follows the Coleman heritage in building a rational choice theory of exchange and power. I believe, however, that in order to complete the chain of *macro–macro* links through *macro–micro* and *micro–macro* links, we have to be equally concerned with the *formalization* of the *macro–micro* link also. Specifically, we have to formalize in rational choice theory (i) social and cultural effects on individual preferences and their initial distribution among actors, (ii) social and cultural constraints on the choices individuals make, (iii) social and cultural effects on individuals’ costs/benefits individuals associate with each possible choice and each possible collective outcome of choices, and (iv) social and cultural effects on the probability structures, which reflect their opportunity structures, that individuals initially estimate — though the probability structures can change as a result of collective action — regarding the relationship between their choices and the possible outcomes of those choices.

This paper is intended to clarify some major principles regarding the determinants of exchange power in *mixed exchange networks*. Mixed exchange networks are networks that include both negative and positive exchange connections (Yamagishi et al., 1988). In modifying and extending the original Coleman model of collective action for applications to mixed exchange networks, this paper takes into account some aspects of elements (i) and (ii) stated above. For (ii), the model takes into account the effects of social-network constraints on choice of exchange partners as in some previous studies described later. For (i), I illustrate it in the application of the model introduced in this paper to dating networks by incorporating the norm of group dates, which generates a specific complementarity of exchange relations. While social and economic institutions must be assumed to be endogenous in the long run, they can be treated as exogenously given in the short run. When Coleman emphasized the theory of *macro–macro* links through the combination of *macro–micro* and *micro–macro* links, he assumed that certain existing institutions impose constraints on individual action. While constraints
may be as strict as the prohibition of certain choice options and the allocation of certain rights among actors, they may also be less restrictive. For example, they may reflect specific societal values and norms in actors’ utility functions. The complementarity between opposite-sex and same-sex relations produced by the group-date norm is just one way to introduce such cultural factors into individual utility functions. Another example of incorporating societal values into utility functions is given in a recent study by Yamaguchi and Ferguson (1995) on fertility. Sociological rational choice theory will greatly benefit from the incorporation of social and cultural factors into individual utility functions inasmuch as the incorporation of these factors permits nontrivial derivation of substantively rich theories and hypotheses.

Specifically, this paper develops further the theory and the model introduced recently by Yamaguchi (1996) for exchange networks as an extension of the modified Coleman model described later. Yamaguchi showed that the concepts of positive and negative exchange connections introduced by Cook and Emerson (1978), Cook et al. (1983), and Yamagishi et al. (1988) can be interpreted by applying the economic notions of complementarity and substitutability, introduced a formal behavioral model of collective action that reflects, in the form of actors’ utility functions, complementarity or substitutability among each actor’s multiple exchange relations, and derived a measure of equilibrium power obtained when actors maximize their utility functions. He also tested predictions from the new power measure with experimental and behavioral–simulational results reported by Cook et al. (1983), Yamagishi et al. (1988), and Skvoretz and Willer (1993).

The model he introduced in his 1996 paper was not applicable to mixed exchange networks, however, because it assumed that each actor’s multiple exchange relations were either all substitutable or all complementary; that is, it did not allow the simultaneous presence of both substitutability and complementarity among the multiple exchange relations of an actor. For example, the model cannot be applied to an exchange network such that among actor A’s three exchange relations with actors B, C, and D, relations A–B and A–C are substitutes but each is a complement to relation A–D. The present paper introduces an extension of the Yamaguchi model that can be applied to mixed exchange networks. This paper also clarifies, using the logic of rational action, some general mechanisms of the determination of the distribution of power in mixed exchange networks. Concrete examples are used for illustrative purposes and the new power measure is applied to them in this paper, but they are used to confirm that the power measure generates characteristics that are logically predicted, and the results of the power measure are not by themselves used to derive a theory. Since experimental research on mixed exchange networks is absent, with Yamagishi et al.’s (1988) study the only exception, this paper does not provide any empirical test of the theory or the model. It is concerned only with the development of a formal rational choice theory of certain exchange networks as in recent studies by Bienenstock and Bonacich (1992), Bonacich (1998), and Montgomery (1996) as well as in studies based on the models developed by extending/modifyng the Coleman model, such as those by Taylor and Coleman (1979), Marsden (1981, 1983), Braun (1994), and Yamaguchi (1996).

The model introduced in this paper is intended to generalize and refine what Yamaguchi (1996) called the modified Coleman model. The original Coleman (1973,
model of collective action assumes a closed system, which has actors, resources, interests and controls as its elements. Actors are interested in resources that are controlled by actors. Actors’ powers are determined by the values of resources they control and the values of resources are determined by demand for them on the part of actors interested in acquiring them. Marsden (1981, 1983) and Braun (1994) introduced modifications of the Coleman model assuming restricted access in networks of exchange.

Taylor and Coleman (1979), however, incorporated a social-network consideration differently, in a model that I refer to as “the modified Coleman model”. This model assumes that actors themselves are the objects of interest and control. It is assumed that actors are interested in other actors (or in exchange relations with other actors), and they exchange control over themselves for control over others in whom they are interested. In this modified model, there is a one-to-one correspondence between actors and resources, and the power of each actor as the subject of control is identical to the value of the actor as the object of control, which is, at the same time, equal to the budget the actor has for the exchange of control. The power of an actor can also be interpreted, under the standardization that each actor initially possesses the same amount (set at 1) of control for supply, as the price needed to acquire unit control over the actor. In other words, the model assumes that the more powerful an actor is, the more difficult it is to obtain control over the actor. This assumption of one-price-for-one-actor, however, means that the model is inadequate to characterize exchange relations where actors exchange goods and services that differ in their prices. Instead, the model is only adequate to characterize situations where actors are interested in other actors and the difficulty of gaining control over an actor is actor-specific — that is, an actor as the object of control is not divisible into elements with different prices to acquire control. The image of such control is the totality of each actor’s time, commitment, affection, etc. Yamaguchi (1996) extended this modified Coleman model for uniformly substitutable and uniformly complementary exchange relations, and the present paper further extends such models for mixed exchange relations.

A more general aim of this paper is to emphasize the importance to the study of social networks of the characteristics of relations of relations, substitutability and complementarity of exchange relations in particular, which modify the effects of network structure on the distribution of exchange power. Here, network structure is assumed to be embedded in the pattern of interest of actors in exchange with other actors, rather than in the pattern of exchange relations among actors. The paper shows that as substitutability among exchange relations increases, actors with substitutes tend to commit to one of the substitutes for exchange, thereby leading to the non-use of certain exchange relations. Hence, this paper assumes that while the pattern of actors’ interest in exchange is exogenously given, the network of exchange relations is an endogenous outcome. It is a collective outcome, generated when actors maximize their utility, of the set of combinations of two individual factors, namely, (1) the structure of each actor’s interest in exchange relations, and (2) the substitutability and complementarity among the multiple exchange relations of each actor reflected by the actor’s utility function. By treating exchange relations as endogenous, this paper sheds some light on some mechanisms of change in exchange relations.
2. A preliminary illustration of mixed exchange networks

Let me first give concrete examples of the mixed exchange networks to be analyzed in this paper. Consider dating networks consisting of two boys and two girls. Dating networks generate typical substitutable exchange relations among an actor’s multiple relations with opposite-sex persons (see Cook et al., 1983 and Yamaguchi 1996 for their illustrative use). We also assume in the example of dating networks analyzed here that there exists a norm of group dates for adolescents such that a boy and a girl must not date by themselves but must each be accompanied by at least one other adolescent of the same sex. As described below, this norm of group dates introduces complementary relations into dating networks. Under the normative constraint of group dates, the dating network consisting of two boys and two girls is minimal in size. The relational patterns for this minimal dating network are not uniform. The dating network consisting of two couples, which may be typical of group dates, is only a special case. In order to identify different patterns of dating networks consisting of two boys and two girls, let us distinguish between two situations for each actor: one in which the actor has undivided interest in opposite-sex persons, and one in which the actor has divided interest in opposite-sex persons. I define the terms as follows: A girl has undivided interest in boys if she is interested in only one of the two boys. On the other hand, a girl has divided interest in boys if she is interested in both boys. (The definitions for boys are similar, with respect to interest in girls.) Generally, when an actor has divided interest, there are infinite possibilities regarding the ratio of relative interest between two opposite-sex persons. Below, I consider two cases regarding this interest ratio, the general case and a special case in which actors have equal interest in two opposite-sex persons. The derived model is applied assuming the special case, although the theoretical discussion is extended to the general case, because certain characteristics hold only for the special case rather than in general.

Ranging over the possibilities of undivided and divided interest in opposite-sex persons for each of the four actors, and over possible objects of interest when actors have undivided interest, yields 15 distinct patterns for dating networks consisting of two boys and two girls, excluding structurally equivalent ones. Structurally equivalent exchange networks here are those obtained either (1) by exchanging positions between boy 1 and boy 2, (2) by exchanging positions between girl 1 and girl 2, or (3) by exchanging positions between the set of two boys and the set of two girls. The 15 distinct networks are given in Fig. 1, where the two boys are labeled as B1 and B2 and the two girls are labeled as G1 and G2. In Fig. 1, the presence of a thick arrow from A to B implies that A has undivided interest and the object of his/her interest is B; the presence of thin arrows from A to the two opposite-sex persons implies that A has divided interest in them. For example, in Pattern 6 of Fig. 1, B1 is interested only in G1; B2 is interested in both G1 and G2; G1 is interested only in B2; and G2 is interested in both B1 and B2.

We assume below that if an actor has divided interest in opposite-sex persons, his/her relations with them are substitutes. This implies that even if an actor is interested in both of two opposite-sex persons, the acquisition of control over (or the affection of) one of them reduces his/her demand for control over (or the affection of)
the other. On the other hand, under the normative constraint of group dating, which requires the accompaniment of a same-sex person, an actor’s relation with an opposite-sex person and his/her relation with a same-sex person are *complementary* because an increase in the frequency of his/her dating with an opposite-sex person increases his/her demand for the accompaniment of a same-sex person. For example, in Pattern 6 of Fig. 1, B1’s relations with G1 and G2 are substitutes and B1’s relation with B2 is a complement to B1’s relation with each of the two girls. Among the 15 patterns depicted in Fig. 1, Patterns 1 through 11 are mixed exchange networks, which thus involve elements of both substitutability and complementarity.

I analyze the determinants of the power distribution for mixed exchange networks below. Among the exchange networks of Fig. 1, Patterns 12 through 15 are not mixed exchange networks because no element of substitutability is involved, and Pattern 1 is a trivial case of equal power. I will therefore concentrate on the remaining 10 patterns. However, the major aim of the analysis presented in this paper is to clarify general principles of the determinants of power distribution; the patterns in Fig. 1 are used for illustration.
Let me raise some questions that will demonstrate the importance of clarifying such
general principles. Let us first consider Patterns 2 and 4 in Fig. 1. In Pattern 2, B1 is
interested only in G1 and the three other actors are interested in both opposite-sex
persons. Like Emerson–Cook’s power-dependence theory (Emerson, 1962; Cook and
Emerson, 1978), the theory and the model introduced below reflect such characteristics
that an actor’s exchange power increases with the extent to which others are interested
in exchange with the actor and are thereby dependent on the actor. It is clear, therefore,
that G1 becomes more powerful than G2 because G1 is the sole object of B1’s interest
in opposite-sex persons and the object of B2’s partial interest, while G2 is only the
object of B2’s partial interest. Which of B1 and B2 becomes more powerful? The
answer is not self-evident. Although the reason will not be clarified until later, B1’s
exchange power becomes somewhat greater than B2’s; this is derived from one of the
general principles described in the next section. In Pattern 4, B1 and B2 are both
interested only in G1, while G1 and G2 are interested in both boys. It is clear that G1
becomes more powerful than G2 and that the powers of two boys are equal (under the
assumption of girls’ equal interest in the two boys). However, which will become more
powerful between boys and girls in Pattern 6 when we measure the power of each sex
by the sum of the powers of the two actors in each group? Again, the answer is not
self-evident. As will be explained later, the girls become somewhat more powerful than
the boys; this is a result of the same principle that explains the difference in power
between B1 and B2 in Pattern 2.

As another example, compare Patterns 5, 6, and 7 in Fig. 1. These three patterns all
represent situations where one of the two boys and one of the two girls have undivided
interest, and the other of the two boys and the other of the two girls have divided
interest. Hence, under the assumption of equal interest when interest is divided, one
of the two boys and one of the two girls each becomes the object of “one-and-a-half
person’s interest,” and the other of the two boys and the other of the two girls each
becomes the object of “half a person’s interest.” It is clear that the former group is
more powerful than the latter, let us therefore call them “the powerful” and “the
powerless”, respectively. The question here is, what explains differences among Pat-
terns 5, 6, and 7 in the extent of power discrepancy between the powerful and the
powerless? The answer to this question may not be self-evident, either. This paper
clarifies a general principle, different from that used to answer the questions about
Patterns 2 and 4, which provides an answer for this question.

The third question is concerned with a general principle regarding the effect of an
increase in substitutability on the distribution of power. In fact, this is one of the main
themes of Yamaguchi’s (1996) study, and a related discussion was also made in
Yamaguchi (1997). He concluded that an increase in substitutability tends to increase
equality in power because the shifting of demand toward exchange with relatively
powerless actors among substitutable exchange partners at least decreases inequality in
power locally, i.e., among partners of each actor whose partners are substitutes.
However, this conclusion was based on an assumption that actors are equally interested
in substitutable partners. The present paper generalizes this finding in two respects. First,
it shows that this tendency, i.e., the trend toward power equality as substitutability
increases, holds for mixed exchange networks under the assumption of actors’ equal
interest in substitutable partners. Second, it shows that when we relax this assumption of equal interest, there are certain situations where an increase in substitutability increases power inequality. This paper also clarifies when and why this occurs.

3. Basic assumptions and three general principles of rational action

Three general behavioral principles are obtained under the assumptions of one-price-for-one-actor and actors’ utility maximization. These principles are introduced by Yamaguchi (1996) and, except for the specificity due to the assumption of actors’ interest in exchange relations rather than in goods and services as the objects of interest and consequent one-price-for-one-actor property of the model, they are well-known behavioral principles in economics. These principles are bases of the theory and the model I further develop in the present paper, but their proofs are omitted because they are shown in Yamaguchi (1996).

Behavioral Principle 1. Suppose that \( x_{ji} \), where \( x_{ji} \geq 0 \) and \( \sum_j x_{ji} = 1 \), is the extent of actor j’s interest in exchange relation with actor i. Then if actor j’s interests in exchange relations are independent, actor j allocates his “budget” \( p_i \) in proportion to his interest in each exchange relation \( x_{ji} \); i.e., he/she allocates \( x_{ji} p_i \) to acquire control over actor i. As a result, the proportion of his/her budget that actor j allocates to actor i does not depend on the “price” \( p_i \) of acquiring unit control over actor i.

Behavioral Principle 2. If actor j’s interests in multiple exchange relations are substitutable, actor j allocates more of his/her budget \( p_i \), than implied by allocation proportional to interest, to relations for which the interest-price ratio, \( x_{ji} / p_i \), is larger. If actor j’s interests \( x_{ji} \) in the objects of control are equal, these correspond to relations with actors whose power \( p_i \) is smaller. If we assume a constant-elasticity-of-substitution (CES) utility function, actor j allocates his/her budget in proportion to \( x_{ji} (x_{ji} / p_i)^{s-1} \), where \( s \) is the parameter for the elasticity of substitution and \( s > 1 \) holds for substitutable relations. In particular, if substitutability is perfect (\( s = \infty \)), then actor j allocates all of his/her budget to be used for substitutable relations to the single relation for which the interest-price ratio is largest.

Behavioral Principle 3. If actor j’s interests in multiple exchange relations are complementary with one-to-one correspondence in quantity, actor j attempts to obtain similar quantities for all elements of complementary objects and therefore allocates \( x_{ji} \), where \( x_{ji} \geq 0 \) and \( \sum_j x_{ji} = 1 \), is the extent of actor j’s interest in exchange relation with actor i. Then if actor j’s interests in exchange relations are independent, actor j allocates his “budget” \( p_i \) in proportion to his interest in each exchange relation \( x_{ji} \); i.e., he/she allocates \( x_{ji} p_i \) to acquire control over actor i. As a result, the proportion of his/her budget that actor j allocates to actor i does not depend on the “price” \( p_i \) of acquiring unit control over actor i.

Behavioral Principle 2. If actor j’s interests in multiple exchange relations are substitutable, actor j allocates more of his/her budget \( p_i \), than implied by allocation proportional to interest, to relations for which the interest-price ratio, \( x_{ji} / p_i \), is larger. If actor j’s interests \( x_{ji} \) in the objects of control are equal, these correspond to relations with actors whose power \( p_i \) is smaller. If we assume a constant-elasticity-of-substitution (CES) utility function, actor j allocates his/her budget in proportion to \( x_{ji} (x_{ji} / p_i)^{s-1} \), where \( s \) is the parameter for the elasticity of substitution and \( s > 1 \) holds for substitutable relations. In particular, if substitutability is perfect (\( s = \infty \)), then actor j allocates all of his/her budget to be used for substitutable relations to the single relation for which the interest-price ratio is largest.

Behavioral Principle 3. If actor j’s interests in multiple exchange relations are complementary with one-to-one correspondence in quantity, actor j attempts to obtain similar quantities for all elements of complementary objects and therefore allocates

---

1 This principle of allocation proportional to one’s interest, which is a characteristic of the Coleman model (1973), is a result of assuming the Cobb–Douglas utility function (Feld, 1977).
more of his/her budget \( p_j \), than implied by allocation proportional to interest, to relations for which the interest–price ratio, \( x_{ji}/p_j \), is nonzero and smaller. If actor \( j \)'s interests \( x_{ji} \) in the objects of control are equal, these correspond to relations with actors whose power \( p \) is larger. If we assume a CES utility function, actor \( j \) allocates his/her budget in proportion to \( x_{ji}(x_{ji}/p_j)^{-c} \), where \( c \) is the parameter for the elasticity of substitution and \( c > 0 \) holds for complementary relations. In particular, if complementarity is perfect \( (c = 0) \), then actor \( j \) allocates his/her budget to be used for complementary objects in proportion to their prices \( p \), so that he/she acquires exactly the same quantity of each complementary element.

4. Formal behavioral model

Although the model can be described for more general mixed exchange networks, I describe it below for dating networks of two boys and two girls, which I use for illustrative purposes. Under the one-price-per actor assumption and the identical amount of initial possession of control, three things are identical below, namely: the exchange power of actor \( j \), which is by definition equal to the total value of controls the actor offers for exchange, namely his/her budget, and the price of the unit control over the actor’s control, which is equated by the assumption that each actor has the same amount of control initially. These three factors are all denoted by \( p_j \) below.

The model assumes the following utility function for each actor \( j \), \( j = 1, 2, 3, \) and \( 4 \).

\[
U_j = \left[ x_{j1} \left( \left( q_{d1,j}^{(c-1)/c} / 2 + q_{s,j}^{(c-1)/c} / 2 \right)^{c/(c-1)} \right)^{1/(1-c)} + x_{j2} \left( \left( q_{d2,j}^{(c-1)/c} / 2 + q_{s,j}^{(c-1)/c} / 2 \right)^{c/(c-1)} \right)^{1/(1-c)} \right]^{1/(1-c)}
\]

(1)

where \( q_{d1,j} \), \( q_{d2,j} \), and \( q_{s,j} \), are, respectively the amounts of control actor \( j \) acquires over the first opposite-sex person \( (G1 \) if actor \( j \) is a boy, \( B1 \) if actor \( j \) is a girl), the second opposite-sex person \( (G2 \) if actor \( j \) is a boy, \( B2 \) if actor \( j \) is a girl), and the same-sex person \( (B2 \) is actor \( j \) is \( B1 \), etc.). We assume here that the total amount of control over each actor held by all actors in the system is standardized to be 1; i.e., \( \sum q_{ij} = 1 \) when \( q_{ij} \) indicates the amount of control actor \( j \) has over actor \( i \).

Utility function \( U_j \) is made up of two hierarchically nested CES functions. The lower level of \( U_j \) consists of two component utilities of control acquired for each pair of the opposite-sex and same-sex persons, \( u_{j1} \equiv \left( q_{d1,j}^{(c-1)/c} / 2 + q_{s,j}^{(c-1)/c} / 2 \right)^{c/(c-1)} \) and \( u_{j2} \equiv \left( q_{d2,j}^{(c-1)/c} / 2 + q_{s,j}^{(c-1)/c} / 2 \right)^{c/(c-1)} \), both of which are CES functions with equal interest weights for the two persons. Since interest in an opposite-sex person and interest in the same-sex person are complementary under the normative constraint of group dates, \( 1 > c \geq 0 \) holds. In particular, if the norm must be followed absolutely and each actor’s interest in the same-sex person is only as accompaniment for dating, complementarity becomes perfect \( (c = 0) \).
The upper level of \( U \) has \( u_{j1} \) and \( u_{j2} \) as its components and is given as \( U_j = (x_{j1}u_{j1}^{(s-1)/s} + x_{j2}u_{j2}^{(s-1)/s})^{s/(s-1)} \), which is also a CES function with \( x_{j1} \) and \( x_{j2} \) as interest weights. If actor \( j \) has undivided interest in the first opposite-sex person, \( x_{j1} = 1 \) and \( x_{j2} = 0 \) hold; if actor \( j \) has undivided interest in the second opposite-sex person, \( x_{j1} = 0 \) and \( x_{j2} = 1 \) hold; and if actor \( j \) has divided interest in opposite-sex persons with equal interest, \( x_{j1} = 0.5 \) and \( x_{j2} = 0.5 \) hold. Parameter \( s \) indicates the extent of substitutability between two opposite-sex persons (or more exactly between two pairs each involving the same-sex person as a companion), and, therefore, \( s > 1 \) holds. We consider below four different situations of substitutability: modest substitutability (\( s = 3 \)), high substitutability (\( s = 9 \)), and very high substitutabilities (\( s = 27, 81 \)).

The model assumes that each actor \( j \) maximizes utility function \( U_j \) under a given budget \( p_j \). Though this \( p_j \) is an endogenous variable, as described below. Because Eq. (1) is a nested CES function, we expect that actor \( j \) first tries to allocate his budget optimally between the first pair and the second pair by following behavioral principle 2 on substitutable relations. Then, for each pair, the actor tries to allocate his/her budget optimally between the opposite-sex person and the same-sex person by following behavioral principle 3 on complementary relations. This is shown more formally in Appendix A.

The actors’ exchange powers, or values of their budgets, also satisfy the following equation.

\[
p_j = \sum p_i q_{ij} \tag{2}
\]

Eq. (2) states that the power of actor \( j \) (\( p_j \)) is equal to the sum over all actors of the product of the power of actor \( i \) (\( p_i \)) times the portion of control that actor \( j \) has over actor \( i \) (\( q_{ij} \)). This assumption is common to the modified Coleman (1973, 1990) model and the Yamaguchi (1996) model.

When actors allocate their budgets, which are equal to their powers, to maximize their utility, the set of \( q_{ij} \) is determined. However, the set of \( q_{ij} \) affects actors’ powers by Eq. (2). Actors thus need to reallocate their new budgets to maximize their utility, which again changes their budgets. Unique equilibrium values of \( p_j \) and \( q_{ij} \), which satisfy both Eq. (2) and a set of equations to be satisfied when actors maximize their utility functions of Eq. (1), can be obtained when \( s < \infty \) and \( c > 0 \), i.e., when both substitutability and complementarity are less than perfect. This equilibrium power does not depend on the initial value of \( p_j \), and this implies that the power distribution is internally uniquely determined in the closed system. We call this equilibrium power the exchange power of the actor. Appendix A describes a set of equations to be satisfied when actors maximize their utility functions, and the iterative estimation method to acquire \( p_j \) and \( q_{ij} \), which satisfy this derived set of equations and Eq. (2).

Since we standardize the amount of “supply” of control over each actor, the power of an actor, which is equal to the “price” of control over the actor, is determined by the

\[ ^2 \text{A unique analytical solution for the equilibrium power may not exist when } s = \infty \text{ or } c = 0. \text{ However, the power distribution always converges when } s \text{ approaches infinity or } c \text{ approaches zero.} \]
demand for control. This characteristic is basically the same as that in economics. The present model and the theory, however, differ significantly from those of competitive markets in economics because the "price" of control over an actor is not exogenous to choices that other actors make and changes interdependently with changes in other actors’ powers and their demand for control over the actor. These characteristics, which are not observable in a competitive market with a large number of actors, produce various phenomena unique to this model of an exchange network. I illustrate some of these unique characteristics using dating networks.

One remark is worth making. As shown in some of the patterns in Fig. 1, some relations between actors are not reciprocated. Hence, direct exchange of control may not take place between certain actors. However, any two persons in any pattern of Fig. 1 are mutually interested at least indirectly within two steps: if, for example, B1 is not interested in G1, he is interested in G2, who is interested in G1 as accompaniment for dating. As in the Coleman model, I assume here that actors can acquire control over others in whom they are interested at least indirectly when there is at least indirect mutual interest. One issue here is that it is quite likely that indirect acquisition of control is less efficient than direct acquisition of control. The model used here, as well as the Coleman model, does not take into account such inefficiency. As shown below, the logic used for identifying who becomes more powerful than whom and why is not affected by this inefficiency. However, some mechanisms generate dependence of power distribution on the extent of this inefficiency. I will discuss this issue in the concluding section.

5. Application and analysis

Table 1 presents the distributions of equilibrium power among four actors for each of the fifteen of dating networks in Fig. 1. The results are obtained by assuming very high but not perfect complementarity ($c = 1/81$) between an opposite-sex person and the same-sex person and four different degrees of substitutability ($s = 3$, $s = 9$, $s = 27$, $s = 81$) between two opposite-sex persons. A small change in the value of $c$, such as replacing it by $c = 1/27$ or $c = 1/243$, among values that indicate a high degree of complementarity affects the results only a very little (the results not presented). I do not employ perfect substitutability ($s = \infty$) or perfect complementarity ($c = 0$) because the unique equilibrium power is obtained for all exchange networks under the condition $s < \infty$ and $c > 0$. The results in Table 1 are based on the assumption that actors with divided interest have equal interest in opposite-sex persons. However, I extend my discussion to the case where actors with divided interest have unequal interest because some characteristics of the results in Table 1 depend on the assumption of equal interest.

Below, I clarify theoretical reasons why we observe specific patterns of power differentiation for each of the 10 mixed exchange networks in Fig. 1 (patterns 2 through 11), other than pattern 1, which is a trivial case of equal power. The results in Table 1 are used only to confirm consistency between logically derived predictions and the mathematical models' predictions.
Table 1
The power distribution of 15 exchange networks as a function of substitutability (c=1/\(s\))

<table>
<thead>
<tr>
<th>Pattern</th>
<th>(s=3)</th>
<th>(s=9)</th>
<th>(s=27)</th>
<th>(s=81)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern 1</td>
<td>0.250 0.250 0.250 0.250 0.191 0.313 0.285 0.211 0.256 0.219 0.379 0.146</td>
<td>0.250 0.250 0.250 0.250 0.211 0.290 0.274 0.225 0.246 0.228 0.380 0.146</td>
<td>0.250 0.250 0.250 0.250 0.229 0.270 0.264 0.236 0.240 0.233 0.381 0.146</td>
<td>0.250 0.250 0.250 0.250 0.240 0.259 0.258 0.243 0.238 0.235 0.381 0.146</td>
</tr>
<tr>
<td>Pattern 2</td>
<td>0.253 0.242 0.304 0.201 0.216 0.284 0.284 0.216 0.250 0.250 0.250 0.250</td>
<td>0.251 0.247 0.284 0.218 0.227 0.273 0.273 0.227 0.250 0.250 0.250 0.250</td>
<td>0.251 0.250 0.267 0.232 0.237 0.263 0.263 0.237 0.250 0.250 0.250 0.250</td>
<td>0.250 0.250 0.258 0.242 0.243 0.257 0.257 0.243 0.250 0.250 0.250 0.250</td>
</tr>
<tr>
<td>Pattern 3</td>
<td>0.250 0.250 0.250 0.250 0.431 0.072 0.423 0.074 0.250 0.250 0.250 0.250</td>
<td>0.250 0.250 0.250 0.250 0.357 0.144 0.354 0.145 0.250 0.250 0.250 0.250</td>
<td>0.250 0.250 0.250 0.250 0.295 0.205 0.295 0.205 0.250 0.250 0.250 0.250</td>
<td>0.250 0.250 0.250 0.250 0.268 0.232 0.268 0.232 0.250 0.250 0.250 0.250</td>
</tr>
<tr>
<td>Pattern 4</td>
<td>0.237 0.237 0.381 0.145 0.206 0.296 0.222 0.276 0.477 0.030 0.462 0.031</td>
<td>0.237 0.237 0.381 0.145 0.221 0.280 0.233 0.266 0.477 0.030 0.462 0.031</td>
<td>0.237 0.237 0.381 0.145 0.234 0.266 0.241 0.259 0.477 0.030 0.462 0.031</td>
<td>0.237 0.237 0.381 0.145 0.242 0.258 0.246 0.254 0.477 0.030 0.462 0.031</td>
</tr>
<tr>
<td>Pattern 5</td>
<td>0.449 0.051 0.449 0.051 0.469 0.027 0.478 0.026 0.481 0.019 0.481 0.019</td>
<td>0.373 0.127 0.373 0.127 0.462 0.031 0.477 0.030 0.481 0.019 0.481 0.019</td>
<td>0.301 0.199 0.301 0.199 0.462 0.031 0.477 0.030 0.481 0.019 0.481 0.019</td>
<td>0.270 0.230 0.270 0.240 0.462 0.031 0.477 0.030 0.481 0.019 0.481 0.019</td>
</tr>
</tbody>
</table>

I use the following simplified expressions in the descriptions below. In Eq. (1), \(x_{31,1}\) and \(x_{31,2} = 1 - x_{31,1}\), for example, indicate, respectively B1’s interest in pair (G1, B2) and in pair (G2, B2) because B1’s dating with a girl must be accompanied by B2. However, I refer to them as B1’s interest in G1 and G2 for simplicity. Secondly, the interest–price ratio, which determines B1’s allocation of his budget between the two substitutes (by applying behavioral principle 2), also needs to be compared between pairs (G1, B2) and (G2, B2). When complementarity between an opposite-sex person and the same-sex person is perfect (c=0), an actor acquires for given allocation exactly the same quantity of control over each; therefore, the unit price of the pair (G1, B2), for example, is the simple sum of the two element prices \(p_{g1} + p_{b2}\). For the results in Table 1, I assumed very high but not perfect complementarity and, therefore, the price of the pair is very close, but not exactly equal, to the simple sum (see formula A12 in Appendix A for the price of a pair with c>0). In the following description, however, I approximate the price of pair (G1, B1), which I denote by \(p_{g1b2}\), by \(p_{g1} + p_{b2}\) for simplicity.
5.1. Pattern 2

In pattern 2, B1 is interested only in G1 and the other three actors are interested in both opposite-sex persons. For simplicity, I assume below that each of the two girls is equally interested in the two boys and consider both equal and unequal interest in the two girls on the part of B2.

Since B1 is interested only in G1 and B2 is interested in both girls, G1 becomes more powerful than G2 because the demand for dating is higher for G1 than for G2. It follows that \( p_{g1} > p_{g2} \) holds. Now since B2 is interested in both G1 and G2, as substitutability increases, B2 tends to allocate all of his power, according to behavioral principle 2 on substitutable relations, to acquiring control over the more advantageous relation of the two. Here, what determines the relative advantage is the relative size of the interest–price ratio between pairs (G1, B1) and (G2, B1), i.e., between \( x_{b2,1}/p_{g1b1} \) and \( x_{b2,2}/p_{g2b1} \). Now if B2 is equally interested in G1 and G2, i.e., if \( x_{b2,1} = x_{b2,2} \), then \( x_{b2,1}/p_{g1b1} < x_{b2,2}/p_{g2b1} \) holds because \( p_{g1} > p_{g2} \) holds. Therefore, as substitutability increases, B2 shifts his budget to acquiring control over G2 rather than G1 and ultimately behaves as if he is interested only in G2 when substitutability becomes perfect \( (s = \infty) \). It follows that as substitutability increases, G2 gains power since she faces more demand for dating from B2, and G1 loses power since she faces less demand for dating from B2, thereby increasing equality of exchange power between G1 and G2. In other words, as substitutability increases, Pattern 2 becomes closer to Pattern 3, and in Pattern 3, the exchange power of G1 and G2 is equal. As shown in the results in Table 1 for Pattern 2 calculated under the assumption that B2 is equally interested in G1 and G2, the difference in exchange power between G1 and G2 becomes smaller as parameter \( s \) for substitutability increases, thereby confirming what we expect theoretically.

Now, let us consider the situation where B2 is not equally interested in G1 and G2. First, we consider for simplicity the situation of perfect substitutability \( (s = \infty) \) where B2 attempts to allocate all of his budget (according to behavioral principle 2) to a more advantageous girl. Which one is it, G1 or G2? The answer depends not only on B2’s relative interest between G1 and G2 but also on the distribution of exchange power in Pattern 4. Though I will describe the results for this pattern later, the distribution of power in Pattern 4 does not depend on the extent of substitutability. As Table 1 shows, the exchange powers of B1, G1, and G2 become 0.237, 0.381, 0.145, respectively, yielding a "price ratio" between pairs (G1, B1) and (G2, B1) of about 0.62 to 0.38.

Suppose now that \( x_{b2,1} \), B2’s interest in G1, is greater than 0.62. Under perfect substitutability, B2 attempts to use all his power to obtain control over the pair whose interest–price ratio is greater. If B2 initially allocates all his power to the (G2, B1) pair, then \( p_{g1} = p_{g2} \) is realized and, since \( x_{b2,1}/p_{g1b1} > x_{b1,2}/p_{g2b1} \) holds, B2 therefore switches the allocation of all his power to the (G1, B1) pair. On the other hand, if B2 initially allocates all his power to the (G1, B1) pair, then the ratio of \( p_{g1b1} / p_{g2b1} \) becomes 0.62 to 0.38 and, since \( x_{b2,1}/p_{g1b1} > x_{b1,2}/p_{g2b1} \) holds, this action is stable. As a consequence, when \( x_{b1,1} > 0.62 \), B2 competes against B1 as if he is solely interested in G1, and the situation becomes equivalent to Pattern 4.

Next suppose \( x_{b2,1} \) \leq 0.50. If B2 initially allocates all his power to the (G2, B1) pair, then \( p_{g1} = p_{g2} \) is realized and, since \( x_{b2,1}/p_{g1b1} \leq x_{b1,2}/p_{g2b1} \) holds, this action is stable.
On the other hand, if B2 initially allocates all his power to the (G1, B1) pair, then the ratio of $p_{g1b1}$ to $p_{g2b1}$ becomes 0.62 to 0.38 and, since $x_{b2,1}/p_{g1b1} < x_{b1,2}/p_{g2b1}$ holds, B2 therefore switches the allocation of all his power to the (G2, B1) pair. As a consequence, when $x_{b1,1} \leq 0.50$, B2 does not compete against B1 and behaves as if he is solely interested in G2, and the situation becomes equivalent to Pattern 3.

Finally, suppose $0.62 > x_{b1,1} > 0.50$ holds: i.e., B2’s interest in G1 is higher than his interest in G2, but not sufficiently high. When B2 allocates all his power to the (G2, B1) pair, $p_{g1} = p_{g2}$ is realized and, since $x_{b2,1}/p_{g1b1} > x_{b1,2}/p_{g2b1}$ holds, it makes B2 switch the object of his control to the (G1, B1) pair; but when B2 allocates all his power to the (G1, B1) pair, then the ratio of $p_{g1b1}$ to $p_{g2b1}$ becomes 0.62 to 0.38 and, since $x_{b2,1}/p_{g1b1} < x_{b1,2}/p_{g2b1}$ holds, it makes B2 switch the object of his control to the (G2, B1) pair. Hence, in this situation the rational choice of one pair always makes the choice of the other pair more advantageous, and therefore, no equilibrium in his budget allocation seems to exist.

In fact, a unique equilibrium always exists when $s < \infty$ but the equilibrium becomes unstable when $s$ is very high in the following sense. If we assume that when actors reallocate their budgets because the prices of objects change, they change them discontinuously from the optimal allocation at step $t$ to the optimal allocation at step $t + 1$, then new budget allocations always jump over the equilibrium allocation. When $s$ is very large, the amount of oscillation in the amount of budget reallocated from one pair to the other pair from step $t$ to step $t + 1$ becomes greater at each step and, therefore, the equilibrium cannot be reached (i.e., the equilibrium is unstable). On the other hand, if $s$ is not very large, a stable equilibrium is obtained because the amount of oscillation in budget allocation becomes smaller at each step, thereby making budget allocations converge to the equilibrium. In particular, when $s = 1$, an equilibrium allocation is attained just by one step because as seen in behavioral principle 1 on independent relations, the optimal proportion of budget to be allocated for each pair does not depend on the prices of objects. On the other hand, if we assume that actors can change their budget allocations smoothly so that they can stop at the equilibrium allocation rather than jump over it, then the equilibrium is stable regardless of $s$ values.

The situation described above occurs because in exchange relations among a small number of actors, “prices” (or exchange powers) may depend heavily on the changes in local demand. When an actor shifts demand for exchange between substitutable persons, the relative power of these persons changes. Because of the assumption of equality between the exchange power of an actor and the “price” of obtaining a unit control over that actor, this change in relative power implies a change in the relative interest–price ratio among substitutes, which affects actors’ rational choices. This kind of situation never occurs in a competitive market with a large number of actors because an actor’s choice does not affect the prices of his/her objects of interest.

I call the situation where an increase in substitutability makes an actor’s rational reallocations of his/her budget less and less optimal under new prices of objects that the actor’s own action generates and, therefore, the rational actor cannot attain an equilibrium budget allocation the situation of ultimate dilemma. I call it “ultimate” because unless substitutability is very high, a stable equilibrium budget allocation between substitutes can be attained.
Thus, B2’s rational choice and its consequence in Pattern 2 can be summarized as follows. As substitutability increases, B2’s behavior converges to one of the following three patterns depending on his relative interest between G1 and G2. (1) When B2’s interest in G1 is sufficiently high (i.e., when $x_{b2,1} > 0.62$), as substitutability increases B2 competes more and more intensively with B1 to obtain control over G1 by reducing his demand for control over G2; (2) When B2’s interest in G1 is less than or equal to his interest in G2 (i.e., when $x_{b2,1} \leq 0.50$), as substitutability increases B2 competes less and less with B1 by shifting more and more of his demand for control from G1 to G2; (3) Finally, when B2’s interest in G1 is higher than that in G2 but not sufficiently high (i.e., when $0.62 > x_{b1,1} > 0.50$), B2 does not commit either to G1 or to G2 ultimately but tries to retain partial control over both of them. This, however, leads to the ultimate dilemma situation and behavioral instability as substitutability becomes very high (under the assumption of discontinuous change in budget allocation).

Now consider the question raised in the introduction: which of the two boys becomes more powerful in Pattern 2 assuming that the girls are equally interested in the two boys? It is clear that the girls’ behavior does not generate a power difference between the boys because they are assumed to be equally interested in B1 and B2. Instead, the difference in power between B1 and B2 is generated by unequal demand for accompaniment between the two boys. By assumption, demand for accompaniment is determined by the principle of complementarity between the opposite-sex relation and the same-sex relation. According to behavioral principle 3 on complementary relations, an actor allocates more of his/her power to get control over the more powerful person between the opposite-sex and the same-sex persons. Since B1 is interested in G1, whose power is greater than B2’s, B1 must allocate more of his power to get dating agreement from G1 than to get accompaniment agreement from B2. On the other hand, while B2 is interested in both G1 and G2, under high substitutability of G1 and G2 and the assumption of equal interest, B2 mainly tries to date G2, whose power is smaller than B1’s. Hence, B2 must allocate more of his power to get B1’s accompaniment agreement than G2’s dating agreement. It follows from this that B2’s dependence on B1 is greater than B1’s dependence on B2, which yields greater power for B1 than for B2. Since this power difference between B1 and B2 is generated by the power difference between G1 and G2, the power difference between B1 and B2 decreases as the power difference between G1 and G2 decreases with the increase of substitutability between the two girls. These characteristics are consistent with the model’s results for Pattern 2 as shown in Table 1.

5.2. Pattern 3 and Pattern 4

Patterns 3 and 4 in Fig. 1 both represent situations where both B1 and B2 have undivided interest in girls and both G1 and G2 have divided interest in boys. Hence, substitutability only affects girls’ behavior here. However, according to behavioral principle 2 on substitutable relations, an actor’s rational choice changes with the extent of substitutability only when the interest–price ratios vary among substitutes. Under the assumption of equal interest in two opposite-sex persons when interest is divided, however, the power [= price] does not differ between the two boys and, therefore, the
interest–price ratios of the two boys are the same for both G1 and G2. Hence, for these
two patterns, the extent of substitutability between the two boys for girls does not affect
the distribution of power, as is confirmed in the results for Patterns 3 and 4 in Table 1.
Furthermore, for Pattern 3, since G1 and G2 are the objects of sole interest on the parts
of B1 and B2, respectively, who have equal power, G1 and G2 also come to have equal
power, yielding the power distribution where all four actors have equal power, as is
confirmed again in the results of Table 1.

On the other hand, in Pattern 4, both B1 and B2 are interested only in G1 and,
therefore, G1 becomes most powerful, G2 becomes least powerful, and the two boys
each get middle-level power. Now, let us consider the question raised in the introd-
uction: Which of the two groups, between boys and girls, obtains more power? This
gender difference in power comes from the difference in the extent of their dependence
on the same-sex person regarding accompaniment for dating. Since the two boys are
interested in G1, G1’s power is greater than that of each boy. It follows from behavioral
principle 3 on complementary relations that each boy must use more of his power to get
dating agreement from G1 than to get accompaniment agreement from the other boy. A
similar situation exists for G1 since each boy is more powerful than G2. On the other
hand, this does not hold for G2. Since G1 is more powerful than each boy, G2 must use
more of her power to get accompaniment agreement from G1 than to get dating
agreement from a boy. That the two boys use more of their power to get control over the
opposite-sex person than the same-sex person (and thereby contribute more to the
exchange power of the opposite-sex person) and one of the girls uses more of her power
to get control over the same-sex person (and thereby contributes more to the exchange
power of the same-sex person) generates greater power for girls than for boys. This is
confirmed in the results for Pattern 4 in Table 1.

5.3. Pattern 5, Pattern 6, and Pattern 7

In Patterns 5, 6, and 7 of Fig. 1, one of the two boys and one of the two girls have
undivided interest in opposite-sex persons, and the other of the two boys and the other of
the two girls have divided interest in opposite-sex persons. Hence, under the assumption
of equal interest when interest in divided, one boy and one girl are each the object of
interest of “one-and-a-half persons” of the opposite sex, and one boy and one girl are
each the object of interest of “one-half person” of the opposite sex. Let us call the
former group the powerful and the latter group the powerless.

In Patterns 5 and 7, the two powerful actors have structurally equivalent positions and
the two powerless actors have structurally equivalent positions. Hence, these patterns
have only two levels of power among the four actors. The difference in power between
the powerful and the powerless is much greater in Pattern 5 than in Pattern 7. In order to
explain this difference, let us review how exchange power is determined. In the model,
an actor’s exchange power (or the price for one unit of control over an actor) is
determined by other persons’ demand for control over the actor (under the standardiza-
tion that the supply of control is the same among all actors in the system). Hence, the
exchange power of an actor is determined not simply by how many others have how
much interest in acquiring control over the actor but also by how powerful (wealthy)
those others are — though power is an endogenous variable here. Hence, let us examine below whether the rank order of power and the extent of difference in power among actors is mainly determined (1) by how many others have how much interest in each actor and (2) by how powerful those others who are interested in each actor are (where the differences between their powers is also approximated by factor 1).

In Pattern 5, the two powerful actors, B1 and G1, are the objects of 100% interest of a powerful opposite-sex actor and the objects of 50% interest of a powerless opposite-sex actor, and the two powerless actors, B1 and G2, are the objects of 50% interest of a powerless opposite-sex actor. Hence, we can expect that the difference in power between the powerful and the powerless is much larger than would follow from the fact that the former are the objects of one-and-a-half persons’ interest and the latter are the objects of one-half person’s interest. The results of Table 1 for Pattern 5 confirm this for situations where substitutability \( s \) is not very large. In Pattern 7, the two powerful actors, B1 and G1, are the objects of 50% interest of a powerful opposite-sex actor and the objects of 100% interest of a powerless opposite-sex actor, and the two powerless actors, B1 and G2, are the object of 50% interest of a powerful opposite-sex actor. Hence, we can expect that the difference in power between the powerful and the powerless is much smaller than would follow from the fact that the former are the objects of one-and-a-half persons’ interest and the latter are the objects of one-half person’s interest. The results of Table 1 for Pattern 7 confirm this. As a result, the difference in power between the powerful and the powerless is much smaller for Pattern 7 than for Pattern 5 at each level of \( s \).

Now, let us consider for Patterns 5 and 7 how the distributions of power changes as substitutability increases under the assumption of equal interest when interest is divided. In Pattern 5, according to behavioral principle 2 on substitutable relations, B2 and G2 shift their demand as substitutability increases toward exchange with each other by reducing their demand for exchange with a powerful opposite-sex person. Hence, as substitutability approaches infinity, Pattern 5 approaches Pattern 12 where two pairs of couples have undivided interest in their partners and all actors have equal power. Similarly in Pattern 7, G1 and B2 each shift their demand for exchange toward a powerless opposite-sex person as substitutability increases, leading ultimately to Pattern 12 as substitutability approaches infinity. These tendencies for Patterns 5 and 7 are observable in the results in Table 1.

Now let us focus on Pattern 6. Between powerful B2 and G1, B2 is the object of 100% interest of powerful G1 and 50% interest of powerless G2, while G1 is the object of 50% interest of powerful B2 and 100% interest of powerless B1. Hence, demand for exchange will be greater for B2 than for G1. Between powerless B1 and G2, B1 is the object of 50% interest of powerless G2 while G2 is the object of 50% interest of powerful B2. Hence, demand for exchange will be greater for G2 than for B1. As a result, we can expect the power order \( B2 > G1 > G2 > B1 \), which is confirmed in the results in Table 1.

In Pattern 6, under the assumption of equal interest, B2 and G2 shift their demand as substitutability increases toward exchange with powerless G2 and B1, respectively. This ultimately makes Pattern 6 converge, as substitutability approaches infinity, to Pattern 13, where actors’ connections of interest are cyclic, \( B1 \rightarrow G2 \rightarrow B2 \rightarrow G1 \rightarrow B1 \), and
the four actors have equal power because their positions are structurally equivalent. Hence, although the patterns to which they converge differ, Patterns 5, 6, and 7 all converge as \( s \) goes to infinity to patterns where all actors have equal power.

However, Patterns 5, 6, and 7 may not converge to Patterns 12, 13, and 12, respectively, as \( s \) goes to infinity if actors who have divided interest in opposite-sex persons have unequal interest in them. A discussion of this topic, however, is omitted for these three patterns and is made in the analysis of Patterns 8 through 11 in the next section.

5.4. Patterns 8, 9, 10, and 11

Patterns 8, 9, 10, and 11 are patterns where only one of the four actors has divided interest in opposite-sex persons and the other three actors have undivided interest in an opposite-sex person. Below, let us examine first whether the rank order of power in these four patterns can be explained by the descriptive criterion used in the previous section, and then secondly how the distribution of power in these patterns change when substitutability increases under the assumption of unequal interest in opposite-sex persons for the actor with divided interest.

In Pattern 8, B1 is the object of one-and-a-half opposite-sex persons’ interest, G1 and G2 are the objects of one opposite-sex person’s interest, and B2 is the object of one-half opposite-sex person’s interest. G1 is the object of interest of more powerful opposite-sex person B1 while G2 is the object of interest of less powerful opposite-sex person B2. Hence, we can expect the power order B1 > G1 > G2 > B2, and the results in Table 1 confirm this. Similarly in Pattern 9, B2 is the object of one-and-a-half opposite-sex persons’ interest, G1 and G2 are the objects of one opposite-sex person’s interest, and B1 is the object of one-half opposite-sex person’s interest. G1 is the object of interest of less powerful opposite-sex person B1 while G2 is the object of interest of more powerful opposite-sex person B2. Hence, we can expect the power order B2 > G2 > G1 > B1, and the results in Table 1 confirm this.

In both Patterns 10 and 11, G1 is the object of two opposite-sex persons’ interest, B1 is the object of one-and-a-half opposite-sex persons’ interest, B2 is the object of one-half opposite-sex person’s interest, and G2 is the object of no opposite-sex person’s interest. Hence, by this fact alone, we can expect the power order G1 > B1 > B2 > G2 and the results in Table 1 for Patterns 10 and 11 confirm this. However, the power of actors who are interested in others also affects the distribution of power. In Pattern 10, the second-ranked B1 is the object of 100% interest of the first-ranked G1 and 50% interest of the fourth-ranked G2, and the third-ranked B2 is the object of 50% interest of the fourth-ranked G2. As a result, we can expect a rather large gap in power between B1 and B2, and the results in Table 1 confirm this. On the other hand, in Pattern 11, the second-ranked B1 is the object of 50% interest of the first-ranked G1 and 100% interest of the fourth-ranked G2, and the third-ranked B2 is the object of 50% interest of the first-ranked G1. As a result, we can expect a rather small gap in power between B1 and B2, and the results in Table 1 confirm this.

We have thus confirmed that for the networks in Fig. 1, the rank order of power and differences in power are primarily determined by how many others have how much interest in each actor as object of control and how powerful those others are. However,
these structural features are not the only factors that determine the distribution of exchange power. The major point of emphasis in this paper is the effect of the presence/absence of substitutability and complementarity and their extent on the distribution of power. As I have described above in the case of Patterns 2 and 4, complementarity affects the distribution of power.

A more important aspect that I focus on in this paper is the effect of the extent of substitutability on the distribution of power. Since only one actor has divided interest in each of Patterns 8 through 11, I consider below how a change in the extent of substitutability between opposite-sex persons affects the power distribution in each case when the actor with divided interest has unequal interest in two opposite-sex persons. As I have already described using Pattern 2, the results depend on the value of the relative interest between the two pairs on the part of the actor with divided interest. If the actor has a sufficiently large interest in an opposite-sex person such that the interest-price ratio is greater for the pair involving that opposite-sex person even when the actor allocates all his/her budget to the pair, then the actor’s demand for control over the pair increases as $s$ increases, leading ultimately to his/her exchange only with the pair. On the other hand, if the actor does not have a sufficiently large interest in either of the two opposite-sex persons, then he/she never commit fully to either of the two opposite-sex persons, but this leads, as $s$ becomes very large, to an ultimate dilemma situation with behavioral instability.

The boundaries in $x$ values between distinct outcomes for Pattern 8, for example, are determined by the “price ratio” between pair (B1, G1) and pair (B2, G1) in Pattern 12, which is obtained when G2 allocates all her power to the (B2, G1) pair, and that in Pattern 14, which is obtained when G2 allocates all her power to the (B1, G1) pair. In Pattern 12, the price ratio between pairs (B1, G1) and (B2, G1) is 0.5 to 0.5 since the power of all actors is equal in this pattern. In Pattern 14, the relative powers of B1, B2, and G1 are 0.477, 0.030, and 0.462, as shown in Table 1, which yield a price ratio between pairs (B1, G1) and (B2, G1) of 0.66 to 0.34. Hence, as $s$ goes to infinity, convergence to Pattern 12 is obtained when $x_{g2,1}$ (G2’s interest in B1) is less than or equal to 0.50, and convergence to Pattern 14 is obtained when $x_{g2,1}$ is greater than 0.66, and an ultimate dilemma situation is obtained when $x_{g2,1}$ is between 0.50 and 0.66.

Similarly, in the case of Pattern 11, Pattern 14 though positions are different from those in Fig. 1) is obtained when G1 allocates all her power to the (B2, G2) pair and Pattern 15 is obtained when G1 allocates all her power to the (B1, G2) pair. The price ratio between pairs (B1, G2) and (B2, G2) is about 0.11 to 0.89 in Pattern 14 and it is about 0.93 to 0.07 in Pattern 15, as shown in Table 1. Hence, as $s$ goes to infinity, convergence to Pattern 14 is obtained when $x_{g1,1}$ (G1’s interest in B1) is less than or equal to 0.11, convergence to Pattern 15 is obtained when $x_{g1,1}$ is greater than 0.93, and an ultimate dilemma situation is obtained when $x_{g1,1}$ is between 0.11 and 0.93, the range of $x$ values where the ultimate dilemma situation is obtained is large in Pattern 11 because G1 is powerful and, therefore, her shift of demand between B1 and B2 affects the relative power between B1 and B2 and, consequently the relative interest-price ratio between pairs (B1, G2) and (B2, G2), greatly.

Table 2 summarizes what pattern each of Patterns 8, 9, 10, and 11 converges to when $s$ goes to infinity depending on the value of $x$, which indicates the relative interest in the
opposite-sex person who is more powerful than the other when $x = 0.5$. As shown in Table 2, Patterns 8, 9, 10, and 11 respectively converge to Patterns 12, 13, 14, and 14 (or their structural equivalents) when $x$ is sufficiently small, and to Patterns 14, 14, 15, and 15 (or their structural equivalents) when $x$ is sufficiently large.

The results in Table 2 show that the ultimate dilemma situation is obtained only in a very narrow range of $x$ in Pattern 10. This occurs because, contrary to Pattern 11, the actor who has divided interest (G2) is powerless and, therefore, her shift of budget between B1 and B2 affects little the relative power between B1 and B2. The results of Table 2 also show that the range of $x$ values for which the actor with divided interest allocates all his/her power to the more powerful opposite-sex person at $s = \infty$ becomes smaller for Pattern 9 than for Pattern 8. This occurs because in Patterns 9, the power of the accompanying same-sex person (G1) for the actor with divided interest (G2) is relatively small and, therefore, the power difference between the two opposite-sex persons greatly affects the difference in prices between the two pairs involving the same-sex person, thereby making it more costly to commit to the more powerful opposite-sex person. On the contrary, in Pattern 8, the power of the accompanying same-sex person (G1) for the actor with divided interest (G2) is large and, therefore, the power difference between the two opposite-sex persons does not affect much the difference in prices between the two pairs involving the same-sex person, thereby making it easier to commit to the more powerful opposite-sex person.

Regarding how the distribution of power changes as the extent of substitutability $s$ increases, Yamaguchi (1996, 1997) concluded that it tends to equalize power because the shifting of demand to less powerful exchange partners, which occurs for each actor having two or more mutually substitutive exchange partners, equalizes the power of those partners locally. However, this conclusion was implicitly based on the assumption that actors with substitutive exchange partners are equally interested in their partners and does not always hold in general. This tendency toward power equalization when the extent of substitutability increases holds, however, for mixed exchange networks when we make the same assumption of equal interest. For the mixed exchange networks in Fig. 1, power is equalized globally as $s$ increases for Patterns 2, 5, 6, 7, 8, and 9. This occurs because each of these six patterns converges, when $s$ goes to infinity under the condition of equal interest, to either Pattern 3, Pattern 12, or Pattern 13, in all of which
power is distributed equally among four actors. On the other hand, power is not equalized globally for Patterns 4, 10, and 11. Pattern 4 is a special case where the value of \( s \) does not affect the distribution of power at all. Pattern 10 converges to Pattern 14 as \( s \) goes to infinity under the condition of equal interest. Since Pattern 14 is characterized by great inequality in power, change in \( s \) values reduces inequality in power very little, as shown in the results of Table 1.

Pattern 11 is another special case. As shown in Table 1, as \( s \) increases, power is equalized only between B1 and B2; the powers of the most powerful actor (G1) and of the least powerful actor (G2) remain stable. Hence, power is equalized only locally here. Pattern 11 is also special because unlike other patterns, it does not converge to any other pattern of Fig. 1 when \( s \) goes to infinity under the condition of equal interest, but converges to a mixture of Patterns 14 and 15. This occurs because \( x = 0.5 \) is within \( x \)'s range of ultimate dilemma situations. When G1 allocates all her power to get control over B2, who is the less powerful of the two boys, as she would when \( s = \infty \), it reverses the relative power between B1 and B2. Under the assumption of a smooth shift of budget allocation, G1 as a rational actor will stop any further transfer of her budget to B2 when B2's power becomes equal to B1's and that becomes the equilibrium state. However, if we assume that G1 shifts her budget discontinuously, then as \( s \) becomes very large, this equilibrium becomes unstable, i.e., an ultimate dilemma situation is obtained. Again, this kind of special situation occurs because a change in an actor's relative demand for control between the two object persons changes their relative power. No similar situation occurs in the competitive market model, where the prices of goods and services do not depend on a single actor's action.

As shown in these examples, under the assumption of equal interest among substitutes, an increase in the extent of substitutability tends to equalize power among actors, though there are situations where the power distribution is either not affected at all or little affected by a change in \( s \) values, and situations where power is equalized only locally and not globally. In no case, however, is there an increase in the inequality of power as \( s \) increases. But what is the result if we do not assume equal interest among substitutes? As we have seen in the results of Table 2, there are situations where an actor with divided interest shifts his/her demand for control toward a more powerful object person when the extent of substitutability increases. This increases the inequality of power at least locally because this shift of demand further increases the power of the more powerful person and decreases the power of the less powerful person among those who are objects of interest for the actor. Hence, we can expect, on average, an increase in the extent of global inequality under such a situation. In general, there will be an increase or a decrease in the local inequality of power among persons who are substitutable objects of control for actors, depending on whether the actors are more likely to shift their demand for control toward more powerful persons or toward less powerful persons as substitutability increases.

6. Discussion

Below, I discuss some implications of the theory and the model introduced in this paper. The following discussion is not concerned with whether the model and the theory
adequately describe the real world. This is an empirical question, which cannot be answered here because experimental results on mixed exchange networks are lacking. The discussion, therefore, is concerned with theoretical implications derived from the model without questioning the basic assumptions of the theory and the model, which in particular include the assumption of rational utility-maximizing action and the one-price-for-one-person property.

This paper has shown that the distribution of power in exchange networks is strongly affected by substitutability and complementarity among multiple exchange relations. As substitutability increases, actors with substitutable object persons of interest tend to either commit to just one of the substitutes or face a dilemma situation ultimately. The dilemma occurs when an actor’s rational choice always makes that choice less attractive than other choices. Since this situation occurs because an actor’s own choice affects the relative exchange powers of his/her object persons of interest, powerful actors, whose influence on the power of their objects of interest is greater than powerless actors, are more likely to face this dilemma. More generally, this paper shows how people make choices among substitutable exchange relations and, therefore, gives some insight into the theory of network formation.

This paper has also shown that although an increase in the extent of substitutability among substitutable object persons of interest likely increases, and never decreases, equality in power among actors when actors with divided interest have equal interest in the substitutable object persons of interest, equality in power may decrease when actors with divided interest have unequal interest in the object persons of interest since they may increase rather than decrease, as substitutability increases, demand for control over more powerful substitutes when they are sufficiently greatly interested in those substitutes.

Below, I would like to discuss subtle differences between the present theory and Emerson–Cook’s power-dependence theory (Emerson 1962; Cook and Emerson, 1978). In the present model and theory, the power of an actor is determined by the demand for control over the actor. It follows that the power of an actor depends on (a) how many others have how much interest in the actor, (b) whether those others are powerful or powerless, (c) what situations those others occupy with respect to substitutes and complements to their relationship with the actor, and (d) how great the extent of such substitutability and complementarity is. The four factors are characteristic of others’ dependence on the actor, and they substantiate Emerson’s more general reference to others’ dependence on the actor as the source of the actor’s power.

What about the effect of an actor’s dependence on others? In the dyadic situation that Emerson (1962) was originally concerned with, an actor’s greater dependence on the other actor directly implies in the present model the actor’s greater demand for control over the other, which increases the other’s power and, consequently, leads to the actor’s power.

---

3Although global structural characteristics of networks also affect the distribution of exchange power (Yamaguchi, 1996), their influence on each actor’s power is mediated locally through factor $b$, which is an endogenous covariate of each actor’s power. The effects of global structural features on power in mixed exchange networks, however, are not analyzed in the present paper because I focus on dating networks made only of two boys and two girls.
loss of relative power compared with the other. This is also consistent with Emerson’s theory.

Then what about the effect of differences among actors in the extent of their dependence on others in a general nondyadic situation? In fact, in the mathematical model introduced in this paper, I assume that actors place all of their interest in others, i.e., \( x_{jj} = 0 \) in the interest matrix, and thereby assume no variation among persons in this respect. As I have shown elsewhere (Yamaguchi 1994, 1996), the distribution of power does not change if we relax this assumption and assume instead that everybody has the same extent of interest in himself/herself, i.e., that \( x_{jj} = \alpha > 0 \) and does not depend on \( j \). However, when the values of \( x_{jj} \), or the extent of noninterest in others (and thereby the relative absence of dependence on others), differ among actors, this affects the distribution of power. Since actors who are less interested in acquiring control over others (i.e., those with larger \( x_{jj} \) values), provide less controls over themselves in the exchange of control, the unit price of control over them becomes higher. If this unit price is applied both to control not offered for exchange and control offered for exchange, the total value of those control over becomes larger, i.e., the power of an actor with a larger \( x_{jj} \) becomes greater. Hence, a smaller extent of dependence on others increases one’s power. If we apply the increased unit price due to a reduction in supply only to control offered for exchange, the total value of that control, or the actor’s exchange power in use, remains the same, however. 5

Next what about the effect of an actor’s pattern of dependence on others assuming that the extent of dependence on others \( x_{jj} \) is the same among actors? Let us consider two cases. One is the effect of the power of the person in whom an actor is interested. The other is the effect of the presence/absence of reciprocated interest. In the first case, the effect on the actor’s own power is rather small, but when a substitute for the object exists it may affect the actor’s power because, as shown in Table 2, the actor is likely to compete against others if his/her primary interest is a powerful other. However, a more clear-cut impact of the power of the object of interest is on the value of the actor’s maximized utility. Generally, there is a correlation between the power (wealth) of a person and the value of his/her maximized utility. At the individual level, greater power leads to the acquisition of more control over objects of interest and thereby greater maximized utility. However, the association between power and maximized utility among persons is much less than perfect. The possession of the same extent of power does not imply the same level of maximized utility. If power is the same, the amount of control acquired over objects of interest depends on the “price” of the objects, or the power of the persons as the objects of interest. The more powerful the object person of interest, the less control can be acquired over that person and therefore the smaller the value of maximized utility. In the standard economic example, a poor person will be much happier when his/her objects of interests are inexpensive goods that he/she can afford than when his/her objects of interest are expensive goods.

---

5 We also need to assume here that interest in oneself is independent in demand from interest in others, i.e., the multiplicative form of this effect as in the Cobb–Douglas function.

5 This conclusion, whose proof is omitted here, is based on the assumption described in footnote 4 that interest in oneself is independent in demand from interest in others.
In the second case, whether the actor’s interest in another actor is reciprocated or not affects the actor’s power little, controlling for the extent of others’ interest in the actor and assuming that actors are interested in each other at least indirectly. This is the case because in the model introduced in this paper, though not in theory, it is assumed that there is no reduced efficiency of indirect acquisition of control. Suppose now, however, that even though the indirect acquisition of control is possible, it is inefficient and imposes an additional transaction cost such that the “price” of obtaining a unit control is higher the more indirect the channel of acquisition. Then, while it does not affect an actor’s power (or wealth) before exchange, the value of acquired controls after exchange becomes less than the value of given controls, and an actor with substitutes will shift his/her demand toward a substitute without involving indirect acquisition of controls as substitutability increases. The inefficiency of indirect acquisition also affects maximized utility: lack of reciprocation and subsequent indirect acquisition of control mean that a smaller amount of control is obtained than would have been possible through direct acquisition and utility is thereby reduced. Hence, an actor’s interest in others who are not interested in the actor is less rewarding than his/her interest in others who are interested in the actor.

As shown in these two examples, actors’ patterns of interest in others have stronger effects on their maximized utility than on their power. In studies of exchange networks, exchange power has been the object of primary interest, and much less attention has been paid to the amount of satisfaction achieved through exchange (i.e., the extent of maximized utility), which also depends on position in the exchange network and on exchange relations with others. The neglect of utility in favor of power may be due primarily to the fact that power is a central subject of interest in sociology whereas the extent of utility attained is a psychological factor. However, if we are concerned with the effects of actors’ patterns of dependence on others in the exchange network, the distinction between power and utility is important.

Acknowledgements
I wish to thank Ronald Burt on his comments on an earlier version of the paper. I am also grateful to Diana Gillooly for her editorial assistance. This paper has grown out of an earlier, shorter paper on mixed exchange networks which appeared in Japanese in *Riron to Hoho* (Yamaguchi, 1999).

Appendix A. Appendix
The derivation of an equilibrium equation is introduced for the case with two boys and two girls. Express the utility of the \( j \)th person, \( U_j \), where \( j = 1, 2, 3, \) and \( 4 \), by

\[
U_j = \left[ x_{j1} \left( \sum_i w_{ij} q_{ij} \right) \left( e^{(c-1)/s} \right) \cdot \left( e/(c-1) \right)^{s/(s-1)} \right]^{s/(s-1)}
\]

\[ + x_{j2} \left( \sum_i w_{ij} q_{ij} \right) \left( e^{(c-1)/s} \right) \cdot \left( e/(c-1) \right)^{s/(s-1)} \right]^{s/(s-1)}
\]

(A1)
where $x_{jl}$ and $x_{j2}$ are as defined in Eq. (1) in the text. The sets of $\{w_{j1}\}$ and $\{w_{j2}\}$ are interest weights for each pair given by the form of 4-by-4 matrix such that $w_{jm}$, $m = 1, 2$, for each person $j$ takes the value 0.5 for $i$ which indicates either the $m$th opposite-sex person or the same-sex person and zero for $i$ which indicates oneself ($w_{jm} = 0$) and the other opposite-sex person.

Eq. (A1) is thus equal to Eq. (1) in the text, but the notation has been changed to make the following description easier. Parameters $q_{ij}$ and $q_{ij'}$ are the quantities of acquired control for cases where the weight $w$ is nonzero. We set $q = 0$ when the corresponding weight is zero.

It is assumed that actor $j$ maximizes $U_j$ under the constraint $p_j = \Sigma_i p_i (q_{ij} + q_{ij'})$. Hence, we maximize,

$$\log(U_j) = \left[ s/(s-1) \right] \log(A) + \lambda_j \left[ p_j - \Sigma_i p_i (q_{ij} + q_{ij'}) \right],$$

(A2)

where $A = x_{jl}(\Sigma_i w_{ij}, q_{ij}^{(e-1)/e}(c/(c-1)(s-1)/s) + x_{j2}(\Sigma_i w_{ij'}, q_{ij'}^{(e-1)/e}(c/(c-1)(s-1)/s)$ and $\lambda_j$ is the Lagrange multiplier. It follows that the first-order derivative conditions are

$$\delta \log(U_j)/\delta \lambda_j = p_j - \Sigma_i p_i (q_{ij} + q_{ij'}) = 0,$$

(A3)

$$\delta \log(U_j)/\delta q_{imj} = (1/A) x_{jmj} \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s) - 1} w_{jmj} q_{imj}^{(e-1)/e} - \lambda_j p_j = 0 \quad \text{for } m = 1, 2.$$  

(A4)

We can rearrange Eq. (A4) to obtain

$$x_{jmj} \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}/A \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}/A \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}$$

$$= \lambda_j p_j q_{imj} \quad \text{for } m = 1, 2.$$  

(A5)

When we sum Eq. (A5) across $i$ for each $m$ and then add the results for the two values of $m$, we obtain $1 = \lambda_j p_j$ and, therefore, $\lambda_j = 1/p_j$ holds.

Then we obtain from Eq. (A5) and $\lambda_j = 1/p_j$ that

$$q_{imj} = \left( p_j/p_i \right) \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}/A \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}/A \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}$$

$$\times x_{jmj} \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}$$

$$= ( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} )^{(c/(c-1)(s-1)/s)}$$

for $m = 1, 2.$  

(A6)

Thus, if $x_{jm} = 0$, $q_{imj} = 0$. When $x_{jm} > 0$, since Eq. (A6) can also be rewritten as

$$A \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)} \left( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} \right)^{(c/(c-1)(s-1)/s)}$$

$$= ( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} )^{(c/(c-1)(s-1)/s)}$$

for $m = 1, 2.$  

(because this should not depend on $i$), we obtain

$$q_{imj}^{(e-1)/e} = \left( ( \Sigma_i w_{jmj} q_{imj}^{(e-1)/e} )^{(c/(c-1)(s-1)/s)} \right)^{-1}$$

(A7)
Hence, when $x_{jm} > 0$, we obtain

$$w_{jm}q_{jm}^{c/(c-1)} \left( \frac{\sum w_{jm}q_{jm}^{c/(c-1)}}{\sum w_{jm}q_{jm}^{c/(c-1)}} \right) = \left( \frac{w_{jm}q_{jm}^{c/(c-1)}}{\sum w_{jm}q_{jm}^{c/(c-1)}} \right),$$

for $m = 1,2$.

Let

$$z_{jm} = \frac{w_{jm}q_{jm}^{c/(c-1)}}{\sum_{k \neq j} w_{jm}q_{jm}^{c/(c-1)}},$$

for $m = 1,2$. (A8)

From this equation and Eq. (A6), we obtain when $x_{jm} > 0$

$$q_{jm} = \left( \frac{p_j}{p_j} \right) z_{jm}R_{jm},$$

for $m = 1,3$. (A9)

where

$$R_{jm} = x_{jm} \left( \frac{\sum w_{jm}q_{jm}^{c/(c-1)/c}}{\sum w_{jm}q_{jm}^{c/(c-1)/c}} \right),$$

for which $R_{jm} = 1$ holds.

Eq. (A9) for $m = 1, 2$ indicates that actor $j$ first allocates his/her “budget” $p_j$ between pair 1 and pair 2 by the ratio of $R_{jm}$ and $R_{jm}$, and then allocates each portion proportionate to $z_{jm}$ to each $i$, where $m = 1$ and 2. Since $z_{jm} = (w_{jm}q_{jm}^{c/(c-1)/c}/\sum_{k \neq j} w_{jm}q_{jm}^{c/(c-1)/c})$ from Eq. (A8), the allocation within the pair proportionate to $z_{jm}$ indicates that behavioral principle 3 on complementary relations holds here.

As will be proved later, $R_{jm}$ can also be expressed as

$$R_{jm} = x_{jm} \left( \frac{\sum w_{jm}q_{jm}^{c/(c-1)/c}}{\sum w_{jm}q_{jm}^{c/(c-1)/c}} \right),$$

for $m = 1,2$. (A11)

Let $P_{jm}$, $m = 1,2$, be defined as

$$P_{jm} = \left( \frac{\sum w_{jm}q_{jm}^{c/(c-1)/c}}{\sum w_{jm}q_{jm}^{c/(c-1)/c}} \right),$$

(A12)

then $R_{jm}$ can be rewritten as

$$R_{jm} = \left( \frac{x_{jm}q_{jm}^{c/(c-1)/c}}{p_jq_{jm}^{c/(c-1)/c}} \right),$$

for $m = 1,2$. (A13)

In fact, $P_{jm}$ defined in Eq. (A12) reflects the price of the pair consisting of the $m$th opposite-sex person and the same-sex person and, therefore, Eq. (A13) indicates that behavioral principle 2 on substitutable relations holds for the initial allocation of $p_j$ to the pairs. Note that when the extent of complementarity is perfect ($c = 0$), Eq. (A12)
indicates that the price of the pair becomes simply the sum of two powers for which \( w_{jmi} > 0 \).

The set of (Eqs. (A8), (A9), (A12) and (A13)) shows that \( q_{i1j} \) and \( q_{i2j} \) can be expressed as functions of \( p_j \), \( p_i \), \( w_{j1i} \), and \( w_{j2j} \) in closed form for a given \( p \).

As the equilibrium condition, however, the set of \( q_{i1j} \) and \( q_{i2j} \) has to satisfy

\[
\sum_j (q_{i1j} + q_{i2j}) = 1 \quad \text{for each } i. \tag{A14}
\]

This leads to an additional equation such that, since \( \sum_i (p_i/p_i)(z_{j1i}R_{j1} + z_{j2i}R_{j2}) = 1 \),

\[\sum_i p_i(z_{j1i}R_{j1} + z_{j2i}R_{j2}) = p_i \text{ holds for each } i.\]

Using Eq. (A8), this equation can be rewritten as:

\[
p_i = \sum_j \left[ \left( \frac{w_{j1i}/p_i^{c-1}}{\left( \sum_k w_{j1k}/p_k^{c-1} \right)} \right) R_{j1} + \left( \frac{w_{j2i}/p_i^{c-1}}{\left( \sum_k w_{j2k}/p_k^{c-1} \right)} \right) R_{j2} \right]. \tag{A15}
\]

In order to obtain the equilibrium power that satisfies this equation, we can use an iterative procedure such that at each \( t \) for given \( p_{i,t} \), we define \( R_{j1,t}, R_{j2,t} \), \( m_{ij,t} \), \( p_{i,t} \), and \( p_{i,t+1} \) in the following order to get the next estimate of \( p_i \).

\[
R_{j1,t+1} = x_{j1} \left( \sum_i w_{j1i}^{c-1} \right)^{(t-1)/(c-1)} \left[ x_{j1} \left( \sum_i w_{j1i}/p_{i,t}^{c-1} \right)^{(t-1)/(c-1)} + x_{j2} \left( \sum_k w_{j2k}/p_k^{c-1} \right)^{(t-1)/(c-1)} \right]. \tag{A16}
\]

\[
r_{i,t+1} = p_{i,t} \left[ \sum_j w_{j1j}/\left( \sum_k w_{j1k}/p_k^{c-1} \right) \right] R_{j1,t+1} + p_{i,t} \left[ \sum_j w_{j2j}/\left( \sum_k w_{j2k}/p_k^{c-1} \right) \right] R_{j2,t+1}. \tag{A17}
\]

\[
q_{i,t+1} = r_{i,t+1}^{1/K}, \tag{A18}
\]

\[
p_{i,t+1} = q_{i,t+1} \sum_j q_{j,t+1}, \tag{A19}
\]

where \( K \geq 1 \) is an arbitrary constant. If convergence is not attained with \( K = 1 \), one should take a larger value of \( K \) to attain convergence — though precision is better for a smaller \( K \) value. Although convergence is slow per iteration, this iterative procedure provides satisfactory convergence for most of the values of \( s \) and \( c \) for nonzero initial power values.

The remainder of the appendix gives the proof of Eq. (A11). It is sufficient if we prove that the ratio \( R_{j1}/R_{j2} \) is equal to the ratio of their numerators, since the denominator is common to the two and \( R_{j1} \) and \( R_{j2} \) sum to 1.
From Eq. (A7), we obtain
\[ B_{jm} = \sum w_{jm} q_{jm}^{(c^{-1})/c} = \left( \frac{p_j}{k_{jm}} \right)^{c-1} \left( \sum w_{jm} / p_i^{c-1} \right) \]
for \( m = 1, 2 \).

If \( x_{j1} > 0 \) and \( x_{j2} > 0 \), then from Eq. (A10) we obtain
\[ R_{j1} R_{j2} = (x_{j1} / x_{j2}) (B_{j1} / B_{j2})^{1/(c/(c-1)(x-1)/x)} \]
\[ = \left( \frac{x_{j1}}{x_{j2}} \right) (k_{j2}/k_{j1})^{(c-1)/s} \left( \sum w_{jm} / p_i^{c-1} \right)^{(c/(c-1)(x-1)/x)} \]
\[ \times \left( \sum w_{jm}^{e_{(s-1)/s}} / p_i^{(c-1)/s} \right)^{(c/(c-1)(x-1)/x)} \]
\[ = \left( \frac{x_{j1}}{x_{j2}} \right) (k_{j2}/k_{j1})^{(c-1)/s} \left( w_{jm} / w_{j2} \right)^{e_{(s-1)/s}} \left( D_{j1} / D_{j2} \right)^{(c-1)/(c-1)/s}. \]

Let \( D_{jm} = (\sum w_{jm} / p_i^{c-1}) \) for \( m = 1, 2 \), then from Eqs. (A9) and (A20), we get
\[ q_{j1}/q_{j2} = \left( \frac{z_{j1}}{z_{j2}} \right) (R_{j1}/R_{j2}) \]
\[ = \left( \frac{x_{j1}}{x_{j2}} \right) (k_{j2}/k_{j1})^{(c-1)/s} \left( w_{jm} / w_{j2} \right)^{e_{(s-1)/s}} \left( D_{j1} / D_{j2} \right)^{(c-1)/(c-1)/s}. \]

On the other hand, from Eq. (A7), we obtain
\[ q_{j1}/q_{j2} = (k_{j2}/k_{j1})^{e_{(s-1)/s}}. \]

From Eqs. (A21) and (A22),
\[ (k_{j2}/k_{j1})^{e_{(s-1)/s}} = (k_{j2}/k_{j1})^{(c-1)/s} = \left( \frac{x_{j1}}{x_{j2}} \right) (D_{j1}/D_{j2})^{(c-1)/(c-1)/s}, \]
and therefore
\[ k_{j2}/k_{j1} = \left( \frac{x_{j1}}{x_{j2}} \right)^{1/e} (D_{j1}/D_{j2})^{(c-1)/(c-1)/s}. \]

From Eqs. (A20) and (A23),
\[ R_{j1}/R_{j2} = \left( \frac{x_{j1}}{x_{j2}} \right)^{1+[(c-1)/(c-1)/s]} \times (D_{j1}/D_{j2})^{(c-1)/(c-1)/s} \times \left( x_{j1}/x_{j2} \right)^{1/(c-1)}/(c-1). \]

Q.E.D.

References

